EMT : Revision of vector contd.

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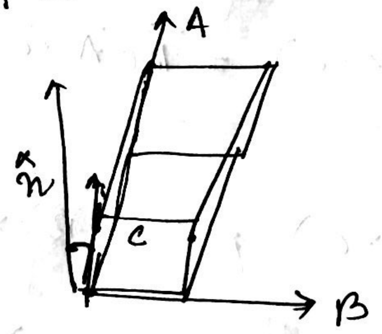
Triple Products

(i) Scalar Triple Product :

$$\vec{A}\cdot(\vec{B}\times\vec{C})$$

Geometrically, $|\vec{A}\cdot(\vec{B}\times\vec{C})|$ is the volume of parallelepiped generated by $\vec{A}$, $\vec{B}$ and $\vec{C}$, since $|\vec{B}\times\vec{C}|$ is the area of the base, and $|A\cos\theta|$ is the altitude



Evidently,

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = \vec{B}\cdot(\vec{C}\times\vec{A}) = \vec{C}\cdot(\vec{A}\times\vec{B}) \quad -(1)$$

* Note that "alphabetical" order is preserved

$$\vec{A}\cdot(\vec{C}\times\vec{B}) = \vec{C}\cdot(\vec{B}\times\vec{A}) = \vec{B}\cdot(\vec{A}\times\vec{C}) \quad -(2)$$

* The non alphabetical triple product have opposite sign

$$\vec{A}\cdot(\vec{C}\times\vec{B}) = -\vec{A}\cdot(\vec{B}\times\vec{C}) \quad -(3)$$

In component form $\vec{A}\cdot(\vec{B}\times\vec{C}) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$

where $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$; $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$
$\vec{C} = C_x\hat{i} + C_y\hat{j} + C_z\hat{k}$ $\quad -(4)$

Note that the dot and cross product can be interchanged

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = (\vec{A}\times\vec{B})\cdot\vec{C} \quad -(5)$$

From equation (1) it can be ~~shown~~ seen. ~~that~~

It will be meaningless expression $(\vec{A}\cdot\vec{B})\times\vec{C}$

$$(\vec{A}\cdot\vec{B})\vec{C} \neq \vec{A}(\vec{B}\cdot\vec{C})$$

(ii) Vector Triple Product: $\vec{A}\times(\vec{B}\times\vec{C})$

$$\boxed{\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})}$$

Note $\vec{A}\times(\vec{B}\times\vec{C}) \neq (\vec{A}\times\vec{B})\times\vec{C}$

$$\vec{A}\times(\vec{B}\times\vec{C}) = -\vec{C}\times(\vec{A}\times\vec{B}) = -\vec{A}(\vec{B}\cdot\vec{C}) + \vec{B}(\vec{A}\cdot\vec{C})$$

Example

$$\vec{A}\times(\vec{B}\times(\vec{C}\times\vec{D}))$$

let $\vec{C}\times\vec{D} = \vec{a}$

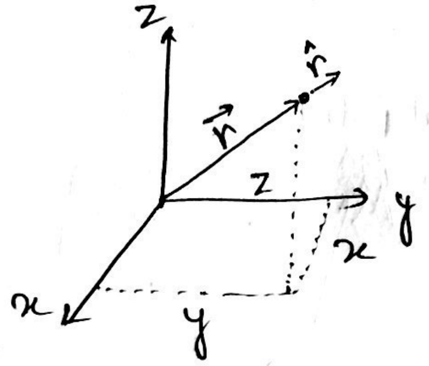
$$\vec{A}\times(\vec{B}\times\vec{a}) = \vec{B}(\vec{A}\cdot\vec{a}) - \vec{a}(\vec{A}\cdot\vec{B})$$

$$= \vec{B}(\vec{A}\cdot(\vec{C}\times\vec{D})) - (\vec{A}\cdot\vec{B})(\vec{C}\times\vec{D})$$

Scanned with CamScanner

Position, Displacement and Separation Vectors

The location of a point in three dimensions can be described by listing its cartesian coordinates (x, y, z). The vector to that point from origin is called the position vector: $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$



Magnitude of $\vec{r} \equiv |\vec{r}| = \sqrt{x^2+y^2+z^2}$

$|\vec{r}|$ is the distance from the origin

and $\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{x\hat{x} + y\hat{y} + z\hat{z}}{\sqrt{x^2+y^2+z^2}}$

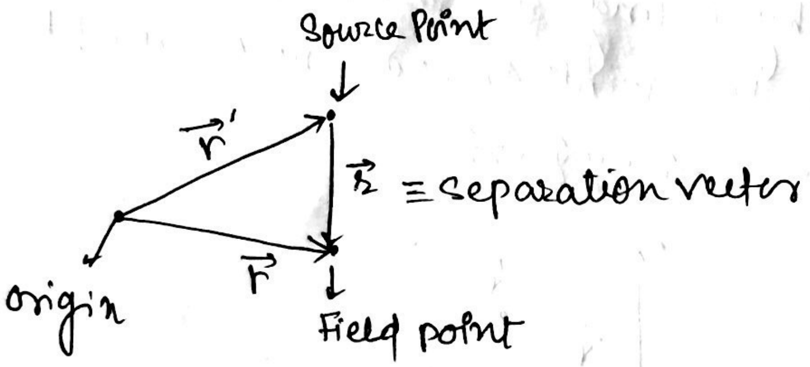
is a unit vector pointing radially outward. ~~The infin~~

The infinitesimal displacement vector from (x, y, z) to $(x+dx, y+dy, z+dz)$ is

$$\vec{dl} = dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}$$

Let a ~~poin~~ source point is at $\vec{r}' \equiv (x'\hat{x} + y'\hat{y} + z'\hat{z})$
and field due to source point is at $\vec{r}$

$$\vec{r} = (x\hat{x} + y\hat{y} + z\hat{z})$$



$$\vec{\mathfrak{r}} \equiv \vec{r} - \vec{r}'$$

$$|\vec{\mathfrak{r}}| = |\vec{r} - \vec{r}'|$$

unit vector in the direction from $\vec{r}'$ to $\vec{r}$ is

$$\hat{\mathfrak{r}} = \frac{\vec{\mathfrak{r}}}{|\vec{\mathfrak{r}}|} = \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|}$$

In cartesian coordinates

$$\vec{\mathfrak{r}} = (x - x')\hat{x} + (y - y')\hat{y} + (z - z')\hat{z}$$

$$|\vec{\mathfrak{r}}| = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$$

$$\hat{\mathfrak{r}} = \frac{(x-x')\hat{x} + (y-y')\hat{y} + (z-z')\hat{z}}{\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}}$$